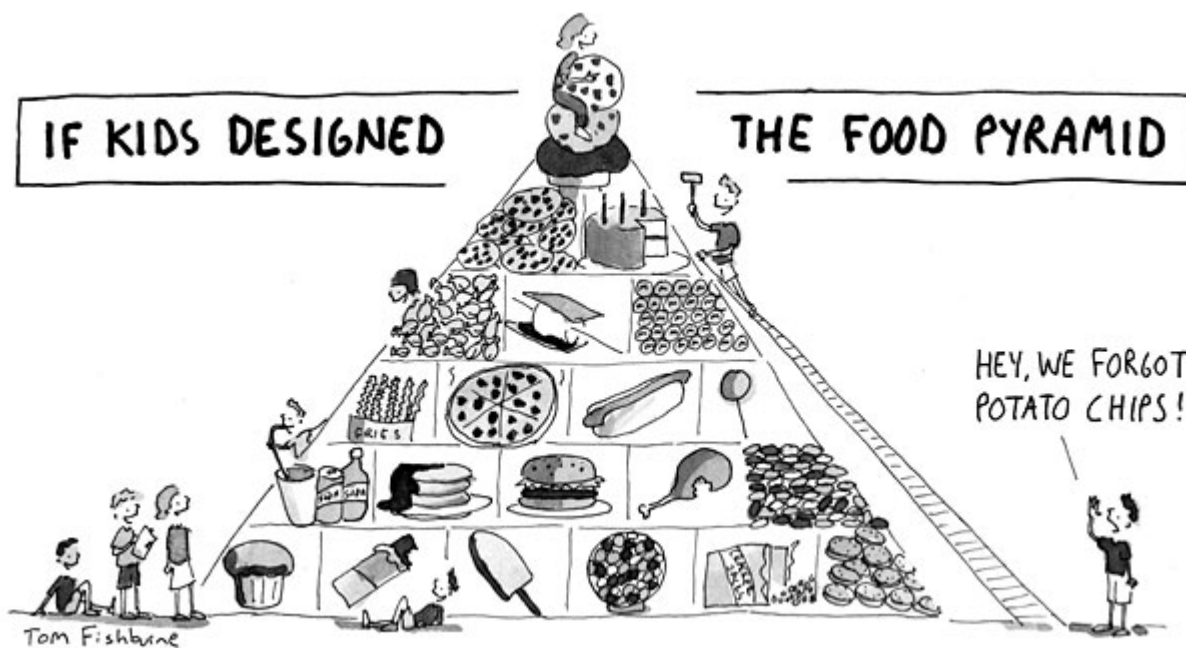# FOOD AND RESTAURANTS



A PROPORTIONAL RELATIONSHIPS UNIT COVERING:

- Ratios, rates, and unit rates
- Understanding and solving proportions
- Proportion word problems
- Scale drawings / Using proportions to determine map distances
- Direct variation / Constant of proportionality
- Coordinate plane
- Unit rate and slope
- Graphs and tables of proportional relationships

"We go together like peas and carrots." – *Forrest Gump*

A ratio makes a comparison.
Forrest has 4 carrots and 8 peas.
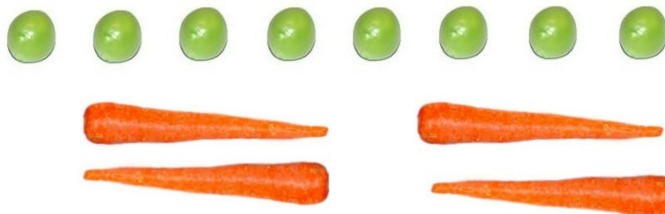
You can write the ratio of carrots to peas
in three different ways:

4 to 8      4:8      $\dfrac{4}{8}$

The ratio of peas to carrots is 8 to 4, 8:4, or $\dfrac{8}{4}$.

You can write a ratio in simplest form the same way you write a fraction in simplest form.  The ratio of carrots to peas in simplest form is:

1 to 2      1:2      $\dfrac{1}{2}$

While not used as frequently, a ratio can compare more than two items.  If there are 3 cakes, 4 cookies, and 5 brownies, you can write this ratio as 3:4:5.

Create a drawing below which represents a ratio of 3 clams to 9 shrimp without drawing exactly 3 clams.

Complete each ratio table.

| Peanut Butter (lb) | 5 | 10 | 15 | | 50 |
|---|---|---|---|---|---|
| Jelly (lb) | 1 | 2 | | 5 | |

| Macaroni (lb) | 1 | 2 | | | 12 |
|---|---|---|---|---|---|
| Cost | $0.50 | $1.00 | $2.00 | $8.00 | |

| Lettuce (lb) | 12 | 24 | 60 | | |
|---|---|---|---|---|---|
| Tomato (lb) | 1 | 2 | | 8 | 10 |

| Pancakes | 10 | 40 | 80 | | |
|---|---|---|---|---|---|
| Syrup (oz) | 1 | | | 11 | 16 |

Complete each ratio table.

1.

| Cookies | 1 | 4 | 7 | | |
|---|---|---|---|---|---|
| Cost | $0.05 | | | $1.20 | $4.00 |

2.

| Wings | 12 | 24 | | | 144 |
|---|---|---|---|---|---|
| BBQ sauce (c) | 4 | | 16 | 28 | |

3.

| Pizzas | 1 | 3 | | 8 | |
|---|---|---|---|---|---|
| Large pepperonis | 7 | | 28 | | 63 |

4.

| Desserts | 1 | | 7 | | |
|---|---|---|---|---|---|
| Little Debbies | 6 | 18 | | 78 | 900 |

Ratios: Express each ratio in simplest form in one of the three ways.

| | Within 10 miles of Mr. Mangham's house there are: 31 McDonalds, 14 Burger Kings, 10 Wendy's, 12 KFC's | | | | |
|---|---|---|---|---|---|
| 5. | McDonalds to BKs | | 6. | Wendys to McDonalds | |
| 7. | KFCs to BKs | | 8. | All 4 to McDonalds | |
| 9. | BKs to McDonalds and KFCs | | 10. | Wendys and KFCs to BKs | |
| 11. | All 4 to The Top 3 | | 12. | The Top 2 to All 4 | |
| 13. | BKs to The Top 2 | | 14. | All 4 to McDonalds, Burger Kings, Wendys, and KFCs | |

| 15. | The ratio of females to males in the restaurant is 6 to 5.  If four females and six males leave, the ratio is 10 to 7.  How many customers originally are in the restaurant? | |
|---|---|---|

A rate is a comparison of two quantities that have different units, such as miles and hours. Rates are often expressed as unit rates, that is, with a denominator of 1 unit.

***Example***:        Ms. Johnsen can cook 135 cookies in 5 hours.

Expressed as a rate: $\dfrac{135 \text{ cookies}}{5 \text{ hours}}$        Expressed as a unit rate : $\dfrac{135 \text{ cookies}}{5 \text{ hours}} = \dfrac{27 \text{ cookies}}{1 \text{ hour}}$

**Rates and Unit Rates: Express each rate as a unit rate.**

|  | Rate | Unit Rate |  | Rate | Unit Rate |
|---|---|---|---|---|---|
| 1. | $\dfrac{105 \text{ peas}}{3 \text{ minutes}}$ |  | 2. | $\dfrac{\$38}{2 \text{ steaks}}$ |  |
| 3. | $\dfrac{2800 \text{ calories}}{7 \text{ days}}$ |  | 4. | $\dfrac{72 \text{ Little Debbies}}{24 \text{ wrappers}}$ |  |
| 5. | $\dfrac{\$375}{5 \text{ days at Kroger}}$ |  | 6. | $\dfrac{\$315}{15 \text{ hamburgers}}$ |  |

**Calculate the unit rates or unit prices for the problems below. Show all your work!**

| | |
|---|---|
| 7. | 5 gallons of water costs $6.25. Find the unit price of each gallon. |
| 8. | Albertsons charges $60 for a 20 pound turkey. Tom Thumb charges $25 for a 10 pound turkey. Which store charges less per pound? |
| 9. | Archie bought 15 gallons of whip cream for $23.85. Joe bought 16 gallons of whip cream for $24.88. Who paid less per gallon? |
| 10. | Andy drove 264 miles using 12 gallons of gas to get to McDonalds. Rita drove 315 miles using 15 gallons of gas to get to Wendys. Who got more miles per gallon? |
| 11. | If a 12 oz. bottle of Mr. Pibb Xtra costs $1.80 and a bottle containing 14 oz. costs $1.96, which is a better buy? |

Express each ratio as a rate and a unit rate. Round to the nearest penny.

|  | Ratio | Rate | Unit Price or Unit Rate |
|---|---|---|---|
| 12. | $2.80 for 64 ounces of orange juice | $\dfrac{\$2.80}{64 \text{ oz.}}$ | $0.04 per ounce |
| 13. | $7.60 for 24 cans of dog food |  |  |
| 14. | $0.50 for 16 ounces of spaghetti |  |  |
| 15. | $1.60 for 6.5 ounces of tuna |  |  |
| 16. | $8.88 for 12 pairs of socks |  |  |

# ManghamMart

ManghamMart competes with three other markets in the neighborhood.  Mr. Mangham wants his prices to be the lowest.  To make this happen, he figures out which of his competitors has the lowest unit rate (or unit price) for an item.  Then he rounds that price to the nearest cent and makes his price 2 cents less per unit.  Find the price of each purchase in ManghamMart.

| Competitors | | | | |
|---|---|---|---|---|
| Item | GoGrocery | FioFruits | MoMarket | ManghamMart |
| Corn | $0.30 each | 4 for $1.00 | 3 for $0.87 | 6 for _____ |
| Pears | 3 lb for $2.67 | 5 lb for $4.59 | 2 lb for $1.89 | 4 lb for _____ |
| Bananas | 4 lb for $1.99 | 3 lb for $1.38 | 2 lb for $0.90 | 1 lb for _____ |
| Apples | 3 for $1.09 | 6 for $2.09 | 3 for $1.00 | 6 for _____ |
| Cantaloupe | 3 for $4.59 | 4 for $6.40 | 2 for $3.09 | 5 for _____ |
| Grapes | 2 lb for $3.49 | 2 lb for $3.44 | 3 lb for $5.19 | 6 lb for _____ |

Help Mrs. Fauatea choose the best bargain for each item.

| | |
|---|---|
| 7. | **Rubber erasers**<br>A.  $0.32 each          B.  Box of 24 for $7.44<br>C.  Buy two for $0.90 and get one free |
| 8. | **Pencils**<br>A.  12 packs of 12 for $17.28          B.  4 pencils for $0.50<br>C.  Buy a box of 12 for $3.12 and get one box free |
| 9. | **Chalk**<br>A.  Box of eight sticks for $0.64          B.  Four packs of 8 sticks for $2.72<br>C.  Bulk box of 1 gross for $10.80 |
| 10. | **Ballpoint pens**<br>A.  $0.27 each          B.  Bulk pack of 100 for $28<br>C.  Buy 2 boxes of 12 for a total of $6.84, get half off the next box |
| 11. | **Calculators**<br>A.  Set of four for $16.92          B.  10-pack for $43<br>C.  Buy one for $8.45, get one free |
| 12. | **Markers**<br>A.  Box of eight for $2.60          B.  12-pack for $4.02<br>C.  Bulk pack of 32 for $13.60, on sale for 20% off |
| 13. | **Protractors**<br>A.  $0.52 each          B.  Box of 24 for $12.36<br>C.  Buy three boxes of four for $7.84, get one box free |

Jeff hikes $\frac{1}{2}$ mile every 15 minutes, or $\frac{1}{4}$ hour.  Lisa hikes $\frac{1}{3}$ mile every 10 minutes, or $\frac{1}{6}$ hour.  How far do they each hike in 1 hour?  2 hours?

1.  Complete the table for Jeff's hike.

| Time (h) | $\frac{1}{4}$ | $\frac{1}{2}$ | $\frac{3}{4}$ | 1 | 2 |
|---|---|---|---|---|---|
| Distance (mi) | $\frac{1}{2}$ | | | | |

2.  Complete the table for Lisa's hike

| Time (h) | $\frac{1}{6}$ | $\frac{1}{3}$ | $\frac{1}{2}$ | 1 | 2 |
|---|---|---|---|---|---|
| Distance (mi) | $\frac{1}{3}$ | | | | |

| 3. | Which hiker walks farther in one hour?  Who is faster? | |
|---|---|---|

While remodeling her kitchen, Angela is repainting.  She estimates that she paints 55 square feet every half-hour.  How many square feet does Angela paint per hour?

Area painted: 55 sq. ft.

Time: $\frac{1}{2}$ hour

$$\frac{55}{\frac{1}{2}} = 55 \div \frac{1}{2} = \frac{55}{1} \cdot \frac{2}{1} = \frac{110 \text{ square feet}}{1 \text{ hour}}$$

| 4. | Paige mows $\frac{1}{6}$ acre in $\frac{1}{4}$ hour.  How many acres does Paige mow per hour? | |
|---|---|---|
| 5. | Greta uses 3 ounces of pasta to make $\frac{3}{4}$ of a serving of pasta.  How many ounces of pasta are there per serving? | |
| 6. | Two pools are leaking.  After 15 minutes, pool A has leaked $\frac{2}{3}$ gallon.  After 20 minutes, pool B has leaked $\frac{3}{4}$ gallon.  Which pool is leaking faster? | |
| 7. | One tank is filling at a rate of $\frac{3}{4}$ gallon per $\frac{2}{3}$ minute.  A second tank is filling at a rate of $\frac{5}{8}$ gallon per $\frac{1}{2}$ minute.  Which tank is filling faster? | |

1. Brandon enters bike races. He bikes $8\frac{1}{2}$ miles every $\frac{1}{2}$ hour. Complete the table to find how far Brandon bikes for each time interval.

| Time (h) | $\frac{1}{2}$ | 1 | $1\frac{1}{2}$ | 2 | $2\frac{1}{2}$ |
|---|---|---|---|---|---|
| Distance (mi) | $8\frac{1}{2}$ | | | | |

Find each unit rate.

| | | | |
|---|---|---|---|
| 2. | Julio walks $3\frac{1}{2}$ miles every $1\frac{1}{4}$ hours. | 3. | Kenny reads $\frac{5}{8}$ of a page in $\frac{2}{3}$ minutes. |

Find each unit rate. Determine which is lower.

| | | |
|---|---|---|
| 4. | Brand A: 240 mg sodium for $\frac{1}{3}$ pickle | Brand B: 325 mg sodium for $\frac{1}{2}$ pickle |
| 5. | Ingredient C: $\frac{1}{4}$ cup for $\frac{2}{3}$ serving | Ingredient D: $\frac{1}{3}$ cup for $\frac{3}{4}$ serving |

The information for two pay-as-you-go cell phone companies is given.

| On Call 3.5 hours: $10 | Talk Time $\frac{1}{2}$ hour: $1.25 |
|---|---|

| | | |
|---|---|---|
| 6. | What is the unit rate in dollars per hour for each company? | |
| 7. | Which company offers the better deal? | |
| 8. | Another company offers a rate of $0.05 per minute. How would you find the unit rate per hour? | |
| 9. | Is the rate for this other company a better deal than OnCall or TalkTime? | |

Amazon sells two packages of protein bars.

| Package | 10-pack of 2.1 oz bars | 12-pack of 1.4 oz bars |
|---|---|---|
| Cost ($) | 15.37 | 15.35 |

| | | | |
|---|---|---|---|
| 10. | Which package has the better price per bar? better price per ounce? | | |
| 11. | Which package do you think is the better buy? | | |

| Pizza Restaurant Name | |
|---|---|
| Owner of the restaurant | |
| Cook | |
| Cashier | |
| Trash person | |
| Hours of operation | |
| Number of pizzas made in 2 hours | |
| Special pizza sale | |
| Amount of trash | |

| | |
|---|---|
| Proportion to solve the number of pizzas made each day | |
| Proportion to solve the cost of the birthday pizzas | |
| Proportion to solve the total amount of trash | |

A proportion is an equation that shows two ratios are equivalent.

**Solve each proportion using the Butterfly Method.**

| | | | | | |
|---|---|---|---|---|---|
| 1. | $\dfrac{8}{15} = \dfrac{m}{45}$ | | 2. | $\dfrac{9}{12} = \dfrac{6}{c}$ | |
| 3. | $\dfrac{5}{p} = \dfrac{3}{9}$ | | 4. | $\dfrac{v}{21} = \dfrac{4}{6}$ | |

**Solve each proportion using the Butterfly Method.  Show all work on a separate sheet of paper.**

| | | | | | |
|---|---|---|---|---|---|
| 5. | $\dfrac{0.4}{m} = \dfrac{2}{4.5}$ | | 6. | $\dfrac{0.18}{0.09} = \dfrac{h}{0.06}$ | |
| 7. | $\dfrac{10}{2.4} = \dfrac{c}{2.64}$ | | 8. | $\dfrac{85.8}{d} = \dfrac{70.2}{9}$ | |
| 9. | $\dfrac{0.6}{1.1} = \dfrac{s}{8.47}$ | | 10. | $\dfrac{2}{3} = \dfrac{x+4}{18}$ | |
| 11. | $\dfrac{4.5}{y+5} = \dfrac{5}{10}$ | | 12. | $\dfrac{0.25}{0.5} = \dfrac{m}{8}$ | |
| 13. | $\dfrac{x}{6} = \dfrac{x+2}{9}$ | | 14. | $\dfrac{12}{x} = \dfrac{27}{x+5}$ | |
| 15. | $\dfrac{x+1}{2} = \dfrac{x+2}{3}$ | | 16. | $\dfrac{2x}{20} = \dfrac{x+20}{50}$ | |
| 17. | $\dfrac{c}{28} = \dfrac{c+4}{35}$ | | 18. | $\dfrac{w-5}{30} = \dfrac{w-7}{20}$ | |

**Use ratios and proportions to answer the following questions.**

| | |
|---|---|
| 19. | Each turnstile can admit 36 people per 2 minutes.  There are 8 turnstiles at entrance A.  How many people can enter the park at this entrance in one minute? |
| 20. | Each student was given an amusement park drink cup for attending as a group.  Each time it is filled, the beverage cost 6 cents per ounce.  How much does it cos to fill your 16 oz. cup four times? |
| 21. | The Whip-A-Round spins five times every 3 seconds.  If you go on the minute-and-a-half ride how many times have you spun around? |
| 22. | The Cliff Drop can accommodate 720 people per hour.  There are 444 people in front of you.  How long until you get to ride? |
| 23. | One gear on the Spiral Coaster turns 7 revolutions every 2 seconds.  How many revolutions are on each minute and a half ride? |

**Use the tables on the next page to write a proportion that can be used to solve the problem.  Solve with the Butterfly Method.  Show all work on a separate sheet of paper.  Label.**

| | |
|---|---|
| 1. | How many calories will a person expend (use) in 3.5 hours of walking? |
| 2. | Fred expended 990 calories playing basketball.  How long did he play? |
| 3. | Laura stretched for 0.25 hours.  How many calories did she expend? |
| 4. | George expended 55 calories doing yard work before he gave up.  How long did he work? |
| 5. | How much would 9 24-packs of water cost? |
| 6. | Chris bought 3.7 pounds of baby back ribs.  To the nearest penny, how much did he spend? |
| 7. | To the nearest penny, how much would 5 12-packs of Coke cost? |
| 8. | Mr. Mangham decided to have a shrimp feast and bought 5.2 pounds of wild shrimp and 4.6 pounds of gulf brown shrimp.  What was his total bill? |
| 9. | How many teaspoons of sugar would be in a 20 oz. glass of chocolate milk? |
| 10. | If Carlee ate 45.5 teaspoons of sugar, all from M&Ms, how many bags did she eat? |
| 11. | How many teaspoons of sugar would be in a 2.4 oz. muffin? |
| 12. | Larry had Jell-O and an ice tea for lunch.  If his total sugar was 34 teaspoons and he had 2 cups of Jell-O, how much tea did he drink? |
| 13. | How many Lucky Charms commercials would a kid see in 6 months?  (nearest whole number) |
| 14. | If Bart Simpson saw 154 Corn Pop commercials, how long has he been watching TV? |
| 15. | How many more Cocoa Puffs commercials than Froot Loops commercials would you expect to see in 2 years? |
| 16. | If Michael watches TV twice as much as a normal kid, in what month will he see his 40th Frosted Flakes commercial? |

Survey of 20 students' favorite and least favorite colors

| Color | Least favorite | Favorite |
|---|---|---|
| Blue | 4 | 7 |
| Green | 3 | 2 |
| Purple | 11 | 8 |
| Red | 1 | 1 |
| Orange | 1 | 2 |

17. If 30 students were surveyed, how many students would choose blue as their least favorite color?

18. If 120 students were surveyed, how many students would choose green as their favorite?

19. If 120 students were surveyed, how many students would NOT choose purple as their favorite?

### Calories Expended per hour for an "average" person

| | | | |
|---|---|---|---|
| Hiking | 370 | Bicycling (>10 mph) | 590 |
| Light gardening | 330 | Swimming | 510 |
| Dancing | 330 | Aerobics | 480 |
| Golf (walking) | 330 | Fast walking | 460 |
| Bicycling | 290 | Heavy yard work | 440 |
| Walking | 280 | Running/jogging | 590 |
| Weight lifting | 220 | Basketball | 440 |
| Stretching | 180 | Tennis | 408 |

### Grocery Store Ads

| | | | |
|---|---|---|---|
| Water, 24 pk | 2 for $5 | Kraft Shedded Cheese, 12 oz. | 3 for $5 |
| Coke, 12 pk. | 3 for $11 | Sirlion Steak | $2.99/lb. |
| Angus Ground Chuck | $1.99/lb. | Baby Back Ribs | $2.99/lb. |
| Salmon Fillets | $4.99/lb. | Peaches | $0.77/lb. |
| Tombstone Pizza | 4 for $10 | Tuna Steaks | $7.99/lb. |
| Wild shrimp | $6.00/lb. | Gulf Brown Shrimp | $7.00/lb. |

### Sugar Content of some foods (teaspoons)

| | | | |
|---|---|---|---|
| Coke, 12oz. | 10 | M&Ms, 1.7 oz bag | 7 |
| Iced Tea, sweetened, 12 oz. | 8 | Muffin, large 4oz. | 6 |
| Chocolate milk, 12 oz. | 6 | Chocolate Chip cookie, 1 oz. | 2 |
| Honey Smacks cereal, 1 oz | 4 | Donut, iced | 6 |
| Popcorn, caramel, cup | 3.5 | Apple pie, 1 piece | 7 |
| Chocolate bar, 1.5 oz. | 6 | Jell-O, 0.5 cup | 4.5 |
| Jam, 1 tbsp. | 2.5 | Syrup, maple 1 tbsp | 3 |

### The 10 Cereals with the Most Ads Targeted toward kids

| | Views | Health Score | | Views | Health Score |
|---|---|---|---|---|---|
| Cinnamon Toast Crunch | 82 | 37 | Frosted Flakes | 58 | 43 |
| Honey Nut Cheerios | 80 | 44 | Fruity/Cocoa Pebbles | 54 | 38 |
| Lucky Charms | 78 | 36 | Reese's Puffs | 52 | 34 |
| Cocoa Puffs | 68 | 39 | Corn Pops | 44 | 36 |
| Trix | 58 | 38 | Froot Loops | 43 | 38 |

Views represent the number of ads seen by an average 6-11yr old over a 15 month period.
The health score is on a scale of 1-100 with over 62 considered healthy.

A scale gives the relationship between the measurements on the scale drawing or model and the measurements of the real object.

Example:  A map shows a scale of 0.5 inches = 6 miles.  The distance between Southlake and Ft. Worth on the map is 3.5 inches.  What is the actual distance?

$$\frac{0.5 \text{ inches}}{6 \text{ miles}} = \frac{3.5 \text{ inches}}{x \text{ miles}}$$
$$0.5x = 6 \bullet 3.5$$
$$\frac{0.5x}{0.5} = \frac{21}{0.5}$$
$$x = 42 \text{ miles}$$

*For all problems below set up a proportion and solve. You may use a calculator for calculations.*
On a set of drawings for a new building, the scale is 0.25 in = 5 ft.  Find the actual length of each room.

| 1. | Lobby | 1.6 in | 2. | CEO Office | 1.35 in |
|---|---|---|---|---|---|
| 3. | Copy Room | 0.55 in | 4. | Secretary Office | 0.6 in |
| 5. | VP Office | 0.9 in | 6. | Library | 1.525 in |
| 7. | Storage Area | 2.1125 in | 8. | Manager Office | 0.625 in |
| 9. | Conference Room | 2.62 in | 10. | Kitchen | 2 in |

On a set of drawings for a new school building, the scale is $\frac{1}{4}$ inch = 2 feet (or 0.25 inches = 2 feet).

Find the missing lengths in the table.

| 11. | Principal's Office | 1.25 in | |
|---|---|---|---|
| 12. | Library | | 16 ft |
| 13. | School Room | 3 in | |
| 14. | Science Lab | 1.5 in | |
| 15. | Cafeteria | | 48 ft |
| 16. | Music Room | 4 in | |
| 17. | Gym | | 56 ft |

| 18. | The greatest distance across Texas is about 660 miles north to south about 790 miles west to east. If you want to fit a map piece of legal paper (8.5 in. × 14 in.) while making it as large as possible, what would be the best scale to use? Set up proportions to prove your answer.<br><br>A   1 in = 125 mi          C   1 in = 75 mi<br>B   1 in = 100 mi          D   1 in = 50 mi | |
|---|---|---|

The Cartesian coordinate system is named after the French mathematician, Rene Descartes (1596-1650).
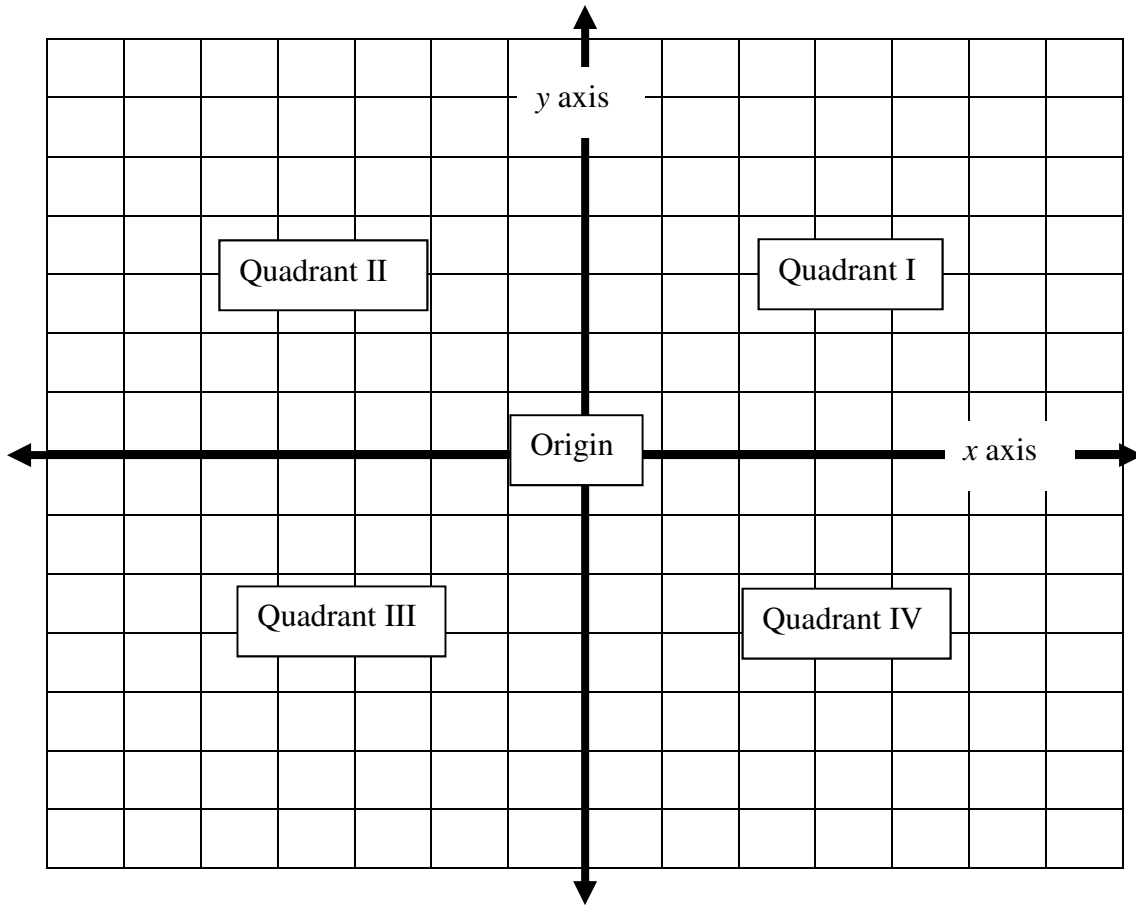


Write your answer in Roman numerals, where possible.
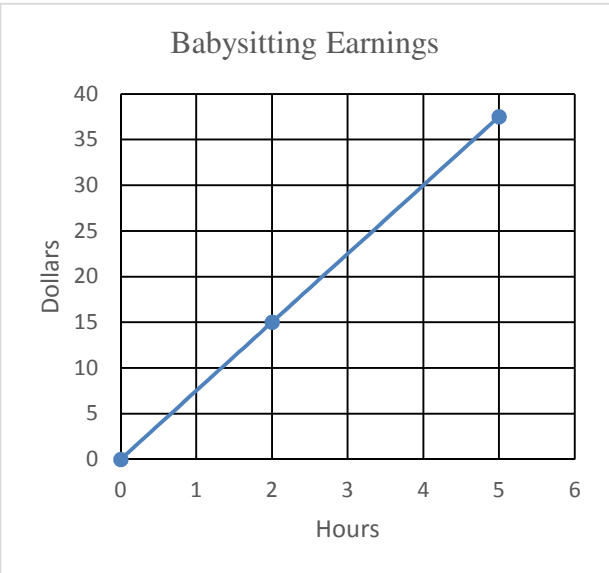
| | | |
|---|---|---|
| 1. | In which quadrant would you find $(-3,2)$? | |
| 2. | In which quadrant would you find $(-5,-8)$? | |
| 3. | In which quadrant would you find $(4,7)$? | |
| 4. | In which quadrant would you find $(8,-10)$? | |
| 5. | In which quadrant would you find $(-13,6)$? | |
| 6. | In which quadrant would you find $(7,-12)$? | |
| 7. | In which quadrant would you find $(-5,-5)$? | |
| 8. | In which quadrant would you find $(13,11)$? | |
| 9. | On which axis would you find $(6,0)$? | |
| 10. | On which axis would you find $(0,8)$? | |
| 11. | On which axis would you find $(0,-13)$? | |
| 12. | Where would you find $(0,0)$? | |

| 1. | First, plot two vertices of a right triangle on the coordinate plane above. Place dot A at $(-2,-5)$ and dot B at $(-4,3)$. | |
|---|---|---|
| 2. | Name the two points that could represent the third vertex of the right triangle. | |
| 3. | Which quadrant is dot A in? | |
| 4. | Reflect dot B across the x-axis. What are the coordinates of the new point B? | |
| 5. | Reflect dot A across the y-axis. What are the coordinates of the new point A? | |
| 6. | Place dot D at (7,4). If dot is translated to the left 5 places and down 9 places where is the new dot D? | |
| 7. | Draw one of the two possible triangles that you could have created in question #2. | |
| 8. | If all three vertices of this triangle are translated up 2 and 5 to the right, what are the coordinates of the three new vertices? | |
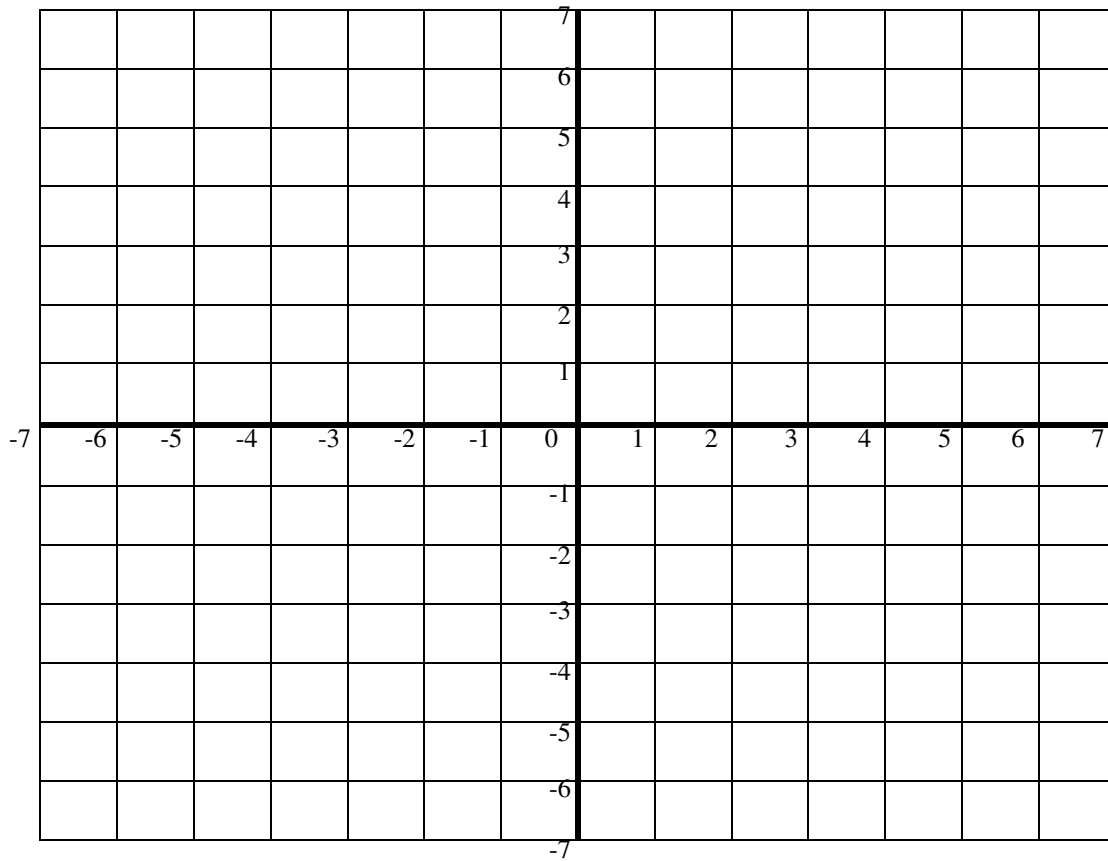
# Properties of Proportional Relationships

- One quantity is a multiple of the other.

- If the first quantity is zero, the second quantity is zero.

- If you double one quantity, the other also doubles.

- The graph of the relationship is a straight line through the origin.

| Constant of Proportionality Summary | |
|---|---|
| **Verbal** | **Table** |
| A babysitter earns $15 in 2 hours and $37.50 in 5 hours. | <table><tr><th>Hours</th><th>$ earned</th></tr><tr><td>2</td><td>15</td></tr><tr><td>5</td><td>37.50</td></tr></table> |
| **Graph** | **Algebraic** |
|  Babysitting Earnings | $y = 7.5x$ |

Many real-world relationships can be described by proportional relationships.  Proportional relationships have special characteristics.

A giant tortoise moves at a slow but steady pace.  It takes the tortoise 3 seconds to travel 10.5 inches.

Complete the table.

| Time (sec) | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Distance (in) | | | 10.5 | | |
| $\dfrac{\text{Distance}}{\text{Time}}$ | | | | | |

| | | |
|---|---|---|
| 1. | How do you think the distance a tortoise travels is related to the time? | |
| 2. | Suppose the tortoise travels for 12 seconds.  Explain how you could find the distance the tortoise travels. | |
| 3. | How would you describe the rate of speed at which a tortoise travels? | |

A **proportional relationship** between two quantities is one in which the rate of change is constant, or one which the ratio of one quantity to the other quantity is constant.

4.  Callie earns money by dog sitting.  Based on the table, is the relationship between the amount Callie earns and the number of days a proportional relationship?

| Number of days | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Amount earned ($) | 16 | 32 | 48 | 64 | 80 |

Find the unit rate for each day and compare the unit rates.  If they are all equal that means the rate is constant and the relationship is proportional.

5.  The table below shows the distance Alison drove one day of her vacation.  Is the relationship between the distance and the time a proportional relationship?  Did she drive at a constant speed?

| Time (h) | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Distance (mi) | 65 | 120 | 195 | 220 | 300 |

Proportional relationships can be written using the equation $y = kx$.

The letter $k$ is called the **constant of proportionality** or **constant of variation**.  These are fancy names for the **unit rate**.

The value of $k$ is represented by the equation $k = \dfrac{y}{x}$.

This proportional relationship is also called a **direct variation**.

<div align="center">

**DIRECT VARIATION:**

$y = kx \quad k \neq 0$

x and y ⟶ variables

k ⟶ constant of variation

</div>

1.  Cashews and their costs are shown in the table below.  Show that the relationship between the number of pounds of cashews and the cost is a proportional relationship.

| Number of pounds | 2 | 3 | 8 |
|---|---|---|---|
| Cost ($) | 19 | 28.50 | 76 |
| $\dfrac{\text{Cost}}{\text{Number of pounds}}$ | | | |

| | | |
|---|---|---|
| 2. | What is the constant of proportionality? | |
| 3. | Let $x$ represent the number of pounds of cashews.  Let $y$ represent the cost in dollars.  Write an equation for the relationship. | |

4.  For a school field trip the number of adults who must accompany the students is listed in the table below.  Show that the relationship between the number of adults and the number of students is a proportional relationship.

| Number of adults | 1 | 3 | 5 |
|---|---|---|---|
| Number of students | 12 | 36 | 60 |
| $\dfrac{\text{Students}}{\text{Adults}}$ | | | |

| | | |
|---|---|---|
| 5. | Write an equation for the relationship. | |

6.  Based on the relationship in the table, is the relationship between time and the number of words typed a proportional relationship?

| Time (min) | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Number of words | 45 | 90 | 135 | 180 |

Activity 5-16: **Writing an Equation for a Proportional Relationship**      Name:

1. Find the constant of proportionality *k*.  Then write an equation for the relationship between *x* and *y*.

| *x* | 2 | 4 | 6 | 8 |
|---|---|---|---|---|
| *y* | 10 | 20 | 30 | 40 |

2. Find the constant of proportionality *k*.  Then write an equation for the relationship between *x* and *y*.

| *x* | 8 | 16 | 24 | 32 |
|---|---|---|---|---|
| *y* | 2 | 4 | 6 | 8 |

Information on three car-rental companies is given.

| **Rent-All** | | | | |
|---|---|---|---|---|
| Days | 3 | 4 | 5 | 6 |
| Total Cost ($) | 55.50 | 74.00 | 92.50 | 111.00 |

| **A-1 Rentals** | **Car Town** |
|---|---|
| The cost *y* of renting a car for *x* days is $10.00 for each half day. | The cost of renting a car from us is just $19.25 per day. |

| | | |
|---|---|---|
| 3. | Write an equation that gives the cost *y* of renting a car for *x* days from Rent-All. | |
| 4. | What is the cost per day of renting a car from A-1? | |
| 5. | Which company offers the best deal?  Why? | |

Steve earns extra money babysitting.  He charges $31.25 for 5 hours and $50 for 8 hours.

| | | |
|---|---|---|
| 6. | Explain why the relationship between how much Steve charges and time is a proportional relationship. | |
| 7. | Write an equation to represent the relationship. | |
| 8. | How much would Steve charge for 3 hours? | |

A submarine dives 300 feet every 2 minutes and 6,750 feet every 45 minutes.

| | | |
|---|---|---|
| 9. | Find the constant rate at which the submarine dives.  Give your answer in feet per minute and in feet per hour. | |
| 10. | Let *x* represent the time of the dive.  Let *y* represent the depth of the submarine.  Write an equation for the proportional relationship using the rate in feet per minute. | |

Created by Lance Mangham, 6th grade math, Carroll ISD

1. The table shows the widths and lengths of United States flags.  Determine whether the data set shows direct variation.

| Width (ft) | 0.5 | 1.25 | 2 | 4.5 | 24 |
|---|---|---|---|---|---|
| Length (ft) | 0.95 | 2.375 | 3.8 | 8.55 | 45.6 |

2. Mr. Mangham is scaling up the side of a cliff and the distance he can climb varies directly with how long he climbs.  How far can he climb in 16 minutes?

   A.  Find the constant of proportionality.

   B.  Write an equation in the form $y = kx$ .

   C.  Predict height he can climb in 16 minutes.

**Mr. Mangham's Hiking Distance**
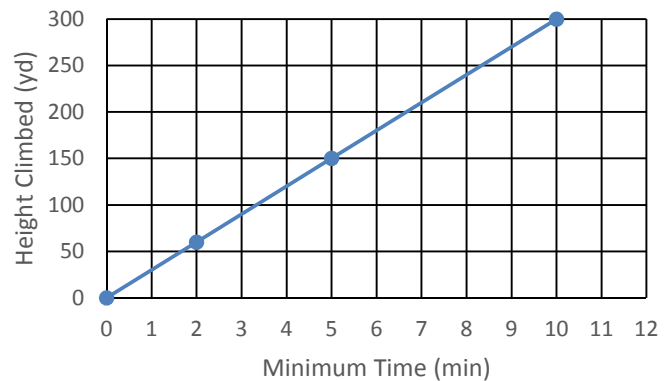


3. Does the equation $y = \dfrac{3}{2}x - 5$ show a direct variation?  Why or why not?

4. One brand of motorcycle uses an oil-to-gasoline ratio as shown in the graph.  The amount of oil that should be added varies directly with the amount of gasoline.

   A.  Write a direct variation equation that describes the relationship.

   B.  Use your equation to determine the amount of oil that should be added to 6.5 gallons of gasoline.
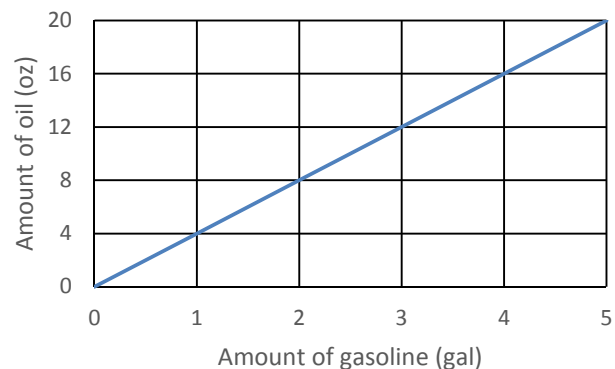
Determine whether the data sets show direct variation. If so, write an equation that describes the relationship in the form $y = kx$.

1.

| Resistance (amps) | 3 | 5 | 6 | 8 |
|---|---|---|---|---|
| Current (ohms) | 4 | 2.4 | 2 | 1.5 |

2.

| Weight (lb) | 8 | 15 | 28 | 47 |
|---|---|---|---|---|
| Dosage (mg) | 160 | 300 | 560 | 940 |

3.

| Games | 20 | 30 | 50 | 160 |
|---|---|---|---|---|
| At-bats | 62 | 93 | 155 | 496 |

4.

| Price ($) | 4 | 12 | 36 | 60 |
|---|---|---|---|---|
| Sales tax ($) | 0.29 | 0.87 | 2.61 | 4.35 |

5. The number of cups in a measure varies directly as the number of tablespoons.

   A. Write a direct variation equation that describes the relationship.

   B. Use your equation to determine the number of cups in 56 tablespoons.
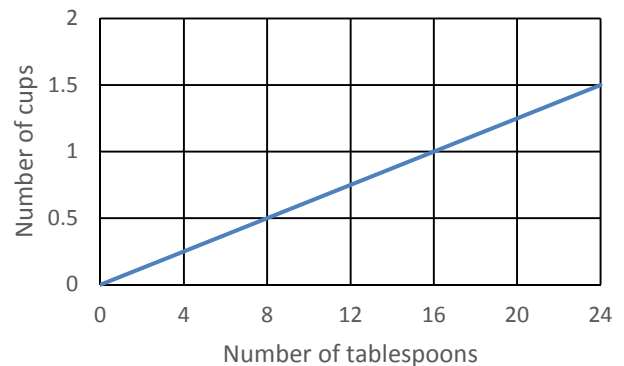


6. The number of calories varies directly with the number of grams of protein.

   A. Write a direct variation equation that describes the relationship.

   B . Use your equation to determine the number of calories from 25 grams of protein.
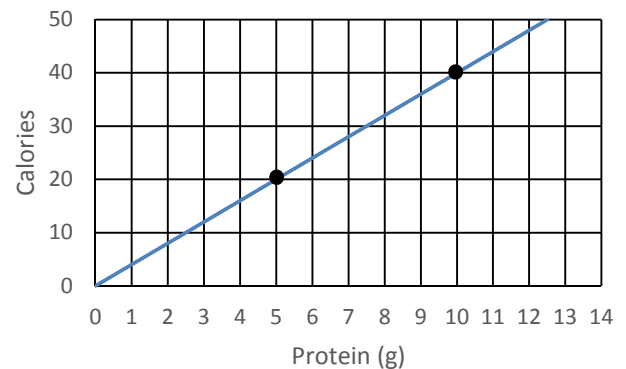


7. Which equations do NOT represent a direct variation?

$$y = \frac{1}{4}x \qquad y = -4x \qquad y = 5x + 1 \qquad y = 6x$$

$$y = -0.53x \qquad y = 4\sqrt{x} \qquad y = -8x + 3 \qquad y = x^3$$

1.  Mischa bought an energy-efficient washing machine.  The amount of water she saves per wash load compared to her old water is shown in the table.

| Number of loads | 1 | 2 | 5 | 6 |
|---|---|---|---|---|
| Water (gal) | 12 | 24 | 60 | 72 |

    A.  Determine whether the relationship is a direct variation.  If so, write an equation that describes the relationship.

    B.  How much water will she save when washing 8 loads?

2.  Sandy wants to build a square garden.  Complete the table for the different side lengths.

| Side length (ft) | 1 | 2 | 3 | 5 | 9 |
|---|---|---|---|---|---|
| Perimeter (ft) | | | | | |
| Area (ft$^2$) | | | | | |

    A.  Does the perimeter of a square vary directly with the side length?  If so, write an equation that describes the relationship.  Explain your answer.

    B.  Does the area of a square vary directly with the side length?  Why or why not?

    C.  Sandy decides to build her garden with a side length of 3 feet.  The border she buys for the perimeter costs $1.99 for a 1.5 foot piece.  The soil she buys covers an area of 3 square feet and costs $4.99 a bag.  How much does Sandy spend on border and soil for her garden?

3.  The three-toed sloth is an extremely slow animal.  Use the graph to write a direct variation equation for the distance $y$ a sloth will travel in $x$ minutes.  How long will it take the sloth to travel 24 feet?

**Speed of a Sloth**
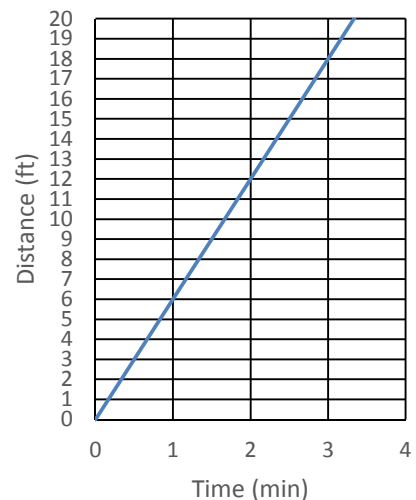


4.  Martin is told that a graph includes the points (2,5) and (4,10).  He says this is a graph of the direct variation $y = 2.5x$.  Do you agree?  Explain.

5.  One graph of a direct variation equation goes through a point A in Quadrant I that is not at the origin.  A second graph of a different variation equation goes through a point that is one unit to the right of A.  Which direct variation equation has a greater constant of variation?  Explain.

Activity 5-20: **Writing an Equation for a Proportional Relationship**     Name: _____

Does the data show direct variation? Write *yes* or *no*. If the data shows direct variation, write an equation to describe the relationship using the constant of proportionality.

1.

| Time (s) | 0.5 | 1 | 1.5 | 2 |
|---|---|---|---|---|
| Distance (ft) | 4 | 16 | 36 | 64 |

_____

2.

| Thickness (in) | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| R-value | 3.14 | 6.28 | 9.42 | 12.56 |

_____

3.

| Hot air (ft$^3$) | 50 | 100 | 500 | 1000 |
|---|---|---|---|---|
| Lift (lb) | 1 | 2 | 10 | 20 |

_____

4.

| Celsius | 5 | 10 | 20 | 30 |
|---|---|---|---|---|
| Fahrenheit | 41 | 50 | 68 | 86 |

_____

**Solve.**

5. The number of pounds in a bag of flour varies directly with the number of ounces. Write a direct variation equation that describes the relationship. Use your equation to determine the number of pounds in 152 ounces.

_____
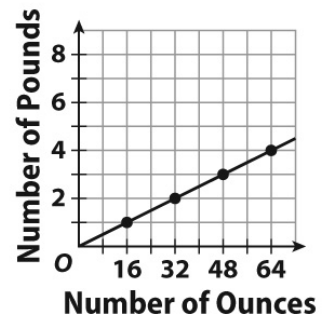


6. The number of pencils for sale varies directly with the number of boxes. Write a direct variation equation that describes the relationship. Use your equation to determine the number of pencils in 14 boxes.

_____
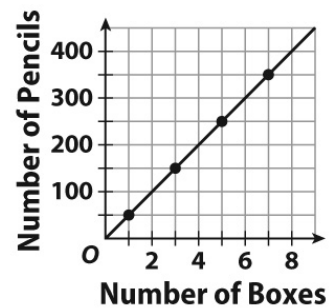
7. The fuel mileage of a hybrid vehicle is shown in the table below.

| **Fuel (gal)** | 6 | 9 | 27 | 40 |
|---|---|---|---|---|
| **Distance (mi)** | 192 | 288 | 864 | 1280 |



  a. Determine if the relationship is a direct variation. If so, write an equation that describes the relationship.

  _____

  b. How many miles can the car drive on 11 gallons of fuel? _____

Activity 5-21: **Proportional Relationships and Graphs**          Name:

Most showerheads that were manufactured before 1994 use 5 gallons of water per minute.  Is the relationship between the number of gallons of water and the number of minutes a proportional relationship?

1.  Complete the table below.

| Time (min) | 1 | 2 | 3 | | 10 |
|---|---|---|---|---|---|
| Water used (gal) | 5 | | | 35 | |
| Water Used / Minute | | | | | |

2.  Based on the table, is this a proportional relationship?

3.  Write the data in the table as ordered pairs  (time, water used).

| (1,5) | (2, ___) | (3, ___) | (___, 35) | (10, ___) |
|---|---|---|---|---|

4.  Plot the ordered pairs.
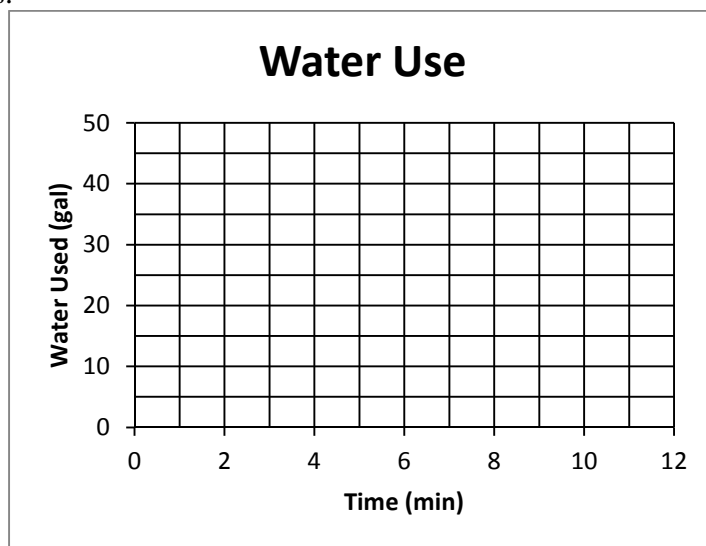


| 5. | If the showerhead is used for 0 minutes, how many gallons of water will be used?  What ordered pair represents this situation?  What is this location called? | |
|---|---|---|
| 6. | If you continued the table to include 23 minutes, would the point (23, 125) be on this graph? | |

*A relationship is a proportional relationship if its graph is a straight line through the origin.*

**There are two situations below: a cleaning company and a bowling game.  One is a proportional relationship and one is not.  Determine which is which and answer the related questions.**

A house cleaning company charges $45 per hour.

| Time (h) | 1 | 2 | 3 | 5 | 8 |
|---|---|---|---|---|---|
| Total Cost ($) | 45 | 90 | 135 | 225 | 360 |

1.  Write the data in the table as ordered pairs and graph the ordered pairs below.
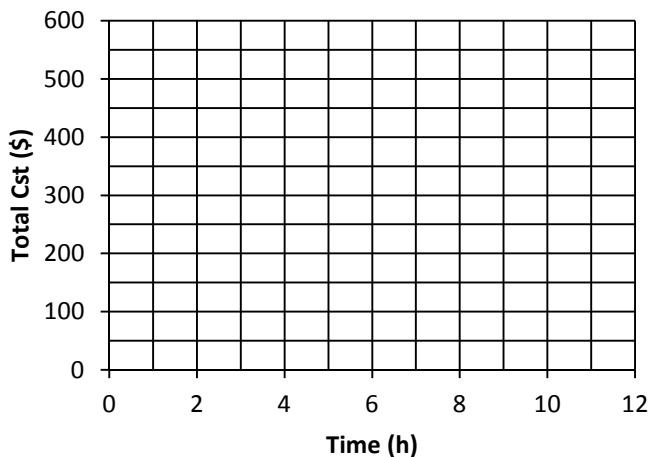


2.  Draw a straight line to connect all of the order pairs in the graph above.  If this is the proportional relationship, write an equation for the relationship.

Jared rents bowling shoes for $6 and pays $5 per bowling game.

| Games | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Total cost ($) | 11 | 16 | 21 | 26 |

3. Write the data in the table as ordered pairs and graph the relationship below.
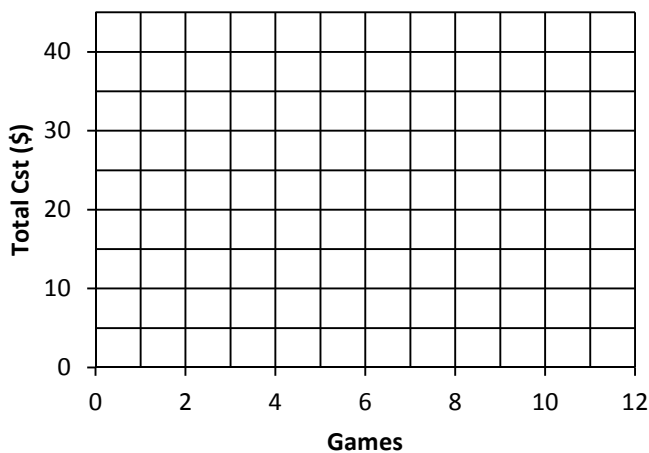


4.  Draw a straight line to connect all of the order pairs in the graph above.  If this is the proportional relationship, write an equation for the relationship.

Complete each table. Explain why the relationship is a proportional relationship.

1. A cashier earns $8 per hour.

| Time (h) | 2 | 4 | | |
|---|---|---|---|---|
| Pay ($) | 16 | | 40 | 72 |

_____

_____

2. Tomatoes cost $0.70 per pound.

| Weight (lb) | 2 | | 6 | 8 |
|---|---|---|---|---|
| Price ($) | 1.40 | 2.10 | | |

_____

_____

Tell whether the relationship is a proportional relationship. Explain your answer.

3.



_____

_____
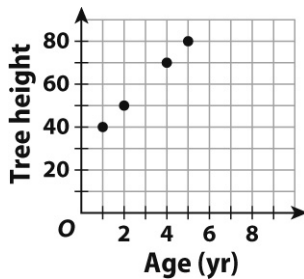
4.



_____

_____
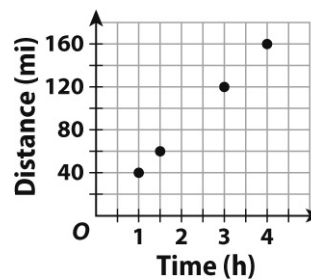
The graph below shows the relationship between time in minutes and the number of miles Darian runs.
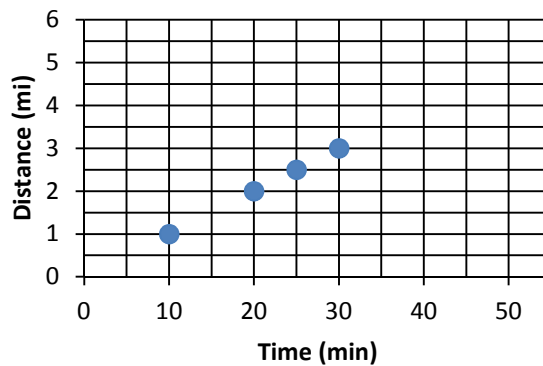


| | | |
|---|---|---|
| 5. | Choose a point on the graph and tell what the point represents. | |
| 6. | What is the constant of proportionality? | |
| 7. | Write an equation in the form $y = kx$. | |
| 8. | Ester runs faster that Darian. Suppose you drew a graph representing the relationship between the time in minutes and distance run for Ester. How would the graph compare to the one for Darian? | |

The graph shows the relationship between the distance a bicyclist travels and the time in hours.
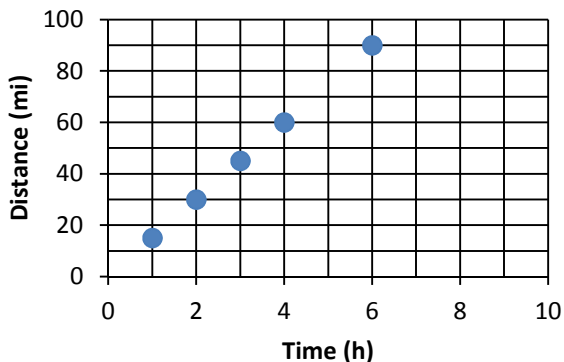


| 1. | What does the point (4,60) represent? | |
|----|---------------------------------------|---|
| 2. | What is the constant of proportionality? | |
| 3. | Write an equation in the form $y = kx$ for this relationship. | |

4-5. Tell whether the relationship is a proportional relationship. Explain why or why not.
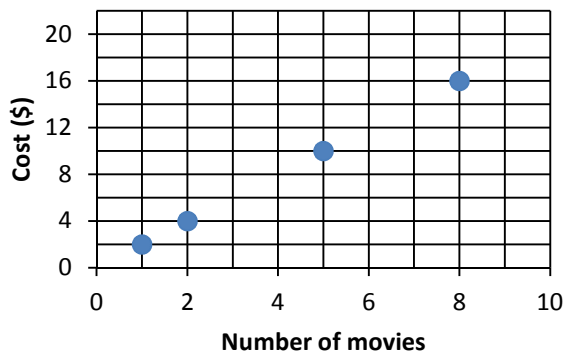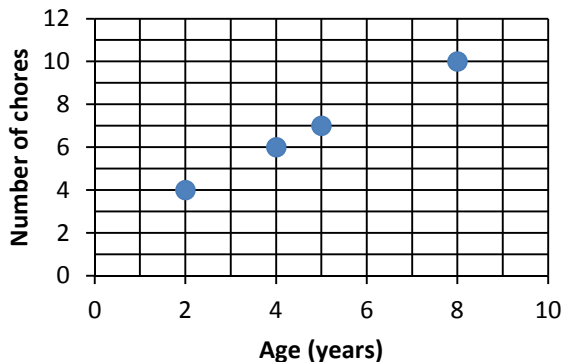


6-7. Write an equation of the form $y = kx$ for the relationship shown in each graph.
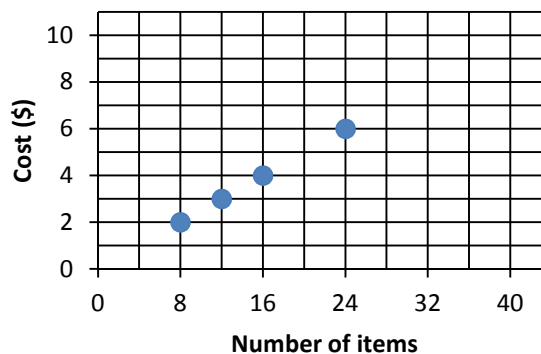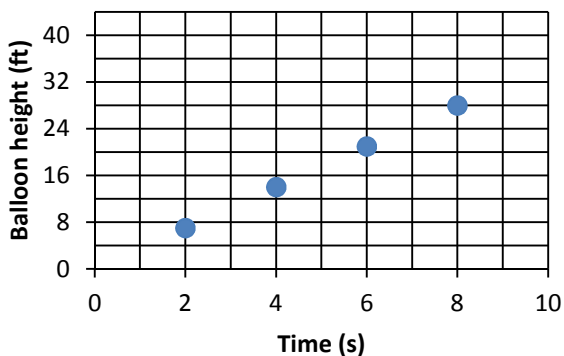
For the following questions use the graph below.  The graph shows the relationship between time and distance run by two horses.
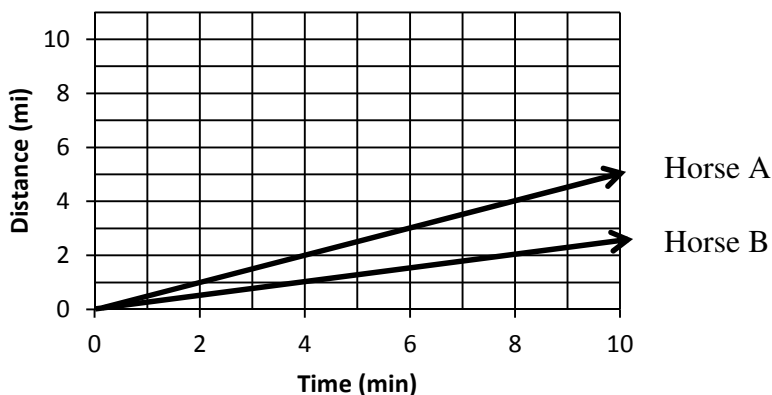


| 1. | Explain the meaning of the point (0,0). | | |
|----|------------------------------------------|---|---|
| 2. | How long does it take each horse to run a mile? | | |
| 3. | Write an equation for the relationship between time and distance for each horse. | | |
| 4. | At the given rates, how far would each horse run in 12 minutes? | | |
| 5. | Draw a line on the graph representing a horse that runs faster than horses A and B. | | |

| 6. | A bullet train can travel at 170 miles per hour.  Will a graph representing distance in miles compared to time in hours show a proportional relationship? | |
|----|---|---|

7.  Bargain DVDs cost $5 each at Mega Movie.  Graph the proportional relationship that gives the cost *y* in dollars of buying *x* bargain DVDs.  Then give an ordered pair on the graph and explain its meaning in the real world context.
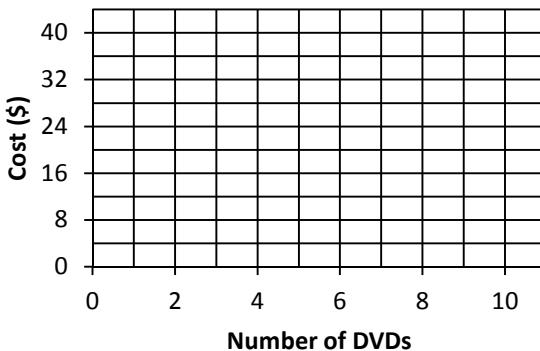
The tables show the distance traveled by three cars.

| Car 1 | |
|---|---|
| Time (h) | Distance (mi) |
| 0 | 0 |
| 2 | 120 |
| 3 | 180 |
| 5 | 300 |
| 6 | 360 |

| Car 2 | |
|---|---|
| Time (h) | Distance (mi) |
| 0 | 0 |
| 5 | 200 |
| 10 | 400 |
| 15 | 600 |
| 20 | 800 |

| Car 3 | |
|---|---|
| Time (h) | Distance (mi) |
| 0 | 0 |
| 1 | 65 |
| 2 | 85 |
| 3 | 105 |
| 4 | 125 |

| | | |
|---|---|---|
| 1. | Which car is not traveling at a constant speed? Explain your reasoning. | |
| 2. | Car 4 is traveling at twice the rate of speed of car 2. How will the table values for car 4 compare to the table values for car 2? | |

| | | |
|---|---|---|
| 3. | Ralph opened a savings account with a deposit of $100. Every month after that, he deposited $20 more. Why is the relationship described not proportional? | |
| 4. | Ralph opened a savings account with a deposit of $100. Every month after that, he deposited $100 more. Is this relationship proportional now? | |
| 5. | How could the situation be changed to make the situation proportional? | |

## Chocolate Chips in Pancakes
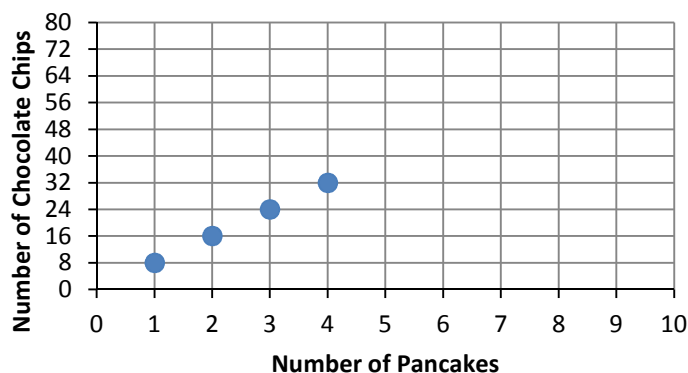


| | | |
|---|---|---|
| 6. | Using the graph, predict the number of chocolate chips in nine pancakes. | |
| 7. | Predict the number of pancakes that would have 48 chocolate chips. | |
| 8. | What does the point (1,8) mean in this situation? | |
| 9. | Write an equation to represent this situation. | |

A **rate of change** is a ratio of the amount of change in the output to the amount of change in the input. For proportional relationships the rate of change is equal to the unit rate and constant of proportionality.

Eve keeps a record of the number of lawns she has mowed and the money she has earned.  Tell whether the rates of change are constant or variable.

|  | Day 1 | Day 2 | Day 3 | Day 4 |
|---|---|---|---|---|
| **Number of lawns** | 1 | 3 | 6 | 8 |
| **Amount earned ($)** | 15 | 45 | 90 | 120 |

Step 1:  Identify the input and output variables.

Input variable ($x$):                    Output variable ($y$):

Step 2:  Find the rate of change.

Day 1 to Day 2:  $\dfrac{\text{change in \$}}{\text{change in lawns}} =$ ——— $=$         Day 2 to Day 3:  $\dfrac{\text{change in \$}}{\text{change in lawns}} =$ ——— $=$

Day 3 to Day 4:  $\dfrac{\text{change in \$}}{\text{change in lawns}} =$ ——— $=$         The rates of change are constant: $\$$____ per lawn.

The table shows the approximate height of a football after it is kicked.  Tell whether the rates of change are constant or variable.

| Time (s) | Height (ft) |
|---|---|
| 0 | 0 |
| 0.5 | 18 |
| 1.5 | 31 |
| 2 | 26 |

Find the rates of change (change in height divided by change in time):

|  |  |  |
|---|---|---|
|  |  |  |

**KEY CONCEPT:**  Rate of change is the **change** in $y$ divided by the **change** in $x$.  This will be the same as $y$ divided by $x$ for proportional relationships, but not for other types of relationships.

You can also use a graph to find rates of change.

The graph shows the distance Nathan bicycled over time.  What is Nathan's rate of change?

1.  Find the rate of change from 1 hour to 2 hours.

$$\frac{\text{change in distance}}{\text{change in time}} =$$

2.  Find the rate of change from 1 hour to 4 hours.

### Nathan's Distance

3.  Find the rate of change from 2 hours to 4 hours.

4.  Recall that the graph of a proportional relationship is a straight line through the origin. Explain whether the relationship between Nathan's time and distance is a proportional relationship.

5.  Does a proportional relationship have a constant rate of change?

6.  Does it matter what interval you use when you find the rate of change of a proportional relationship?

# SLOPE

When the rate of change of a relationship is constant, every segment of its graph has the same steepness, and the segments together form a line.  The constant rate of change is called the **slope** of the line.

The slope of a line is the ratio of the change in y-values (rise) for a segment of the graph to the corresponding change in x-values (run).

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

rise                                    rise

run                                     run

For *proportional* relationships:

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \text{Unit Rate} = \text{Constant of Proportionality} = k = m$$
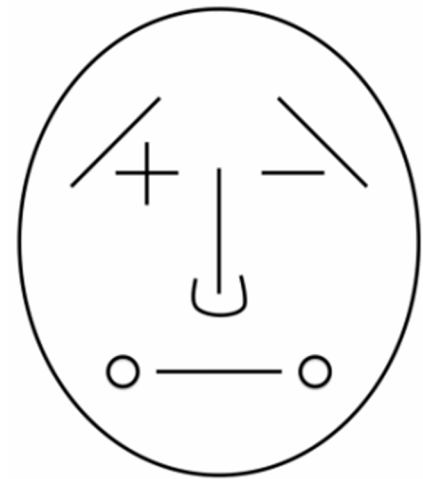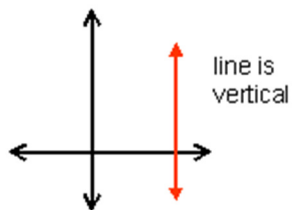
## Positive Slope

line rises
from left
to right

## Negative Slope

line falls
from left
to right

**MR. SLOPE GUY**

## Slope of 0

line is
horizontal

## Undefined Slope

line is
vertical

SLOPE
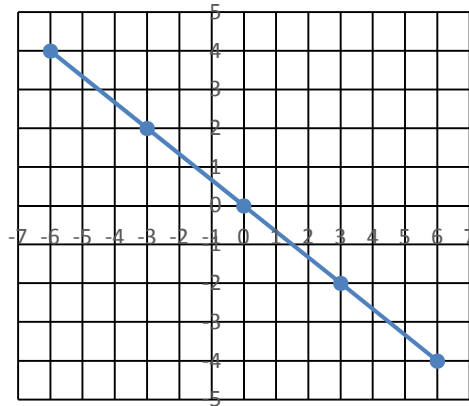
You have to RISE
before you
can RUN!

TRACK MEET
TODAY

Find the slope of the line.

Step 1: Choose two points on the line.

Step 2: Find the change in y-values (rise) and the change in x-values (run) as you move from one point to the other.

$$\text{rise} = +2 \qquad \text{run} = -3$$

Step 3:   $\text{Slope} = \dfrac{\text{rise}}{\text{run}} = \dfrac{2}{-3} = -\dfrac{2}{3}$



The graph shows the rate at which water is leaking from a tank. The slope of the line gives the leaking rate in gallons per minute.

Rise =

Run =

Rate of leaking = _____ gallons per minute



Leaking Tank

Prove whether the rates of change are constant or variable. Show all work.

1. Building measurements _____

| Feet | 3 | 12 | 27 | 75 |
|---|---|---|---|---|
| Yards | 1 | 4 | 9 | 25 |

2. Computers sold _____

| Week | 2 | 4 | 9 | 20 |
|---|---|---|---|---|
| Number Sold | 6 | 12 | 25 | 60 |

3. Distance an object falls _____

| Time (s) | 2 | 3 | 5 | 8 |
|---|---|---|---|---|
| Distance (ft) | 16 | 21 | 31 | 46 |

4. Cost of sweaters _____

| Number | 2 | 4 | 7 | 9 |
|---|---|---|---|---|
| Cost ($) | 38 | 76 | 133 | 171 |

Find the slope (or rate of change) between each pair of points.

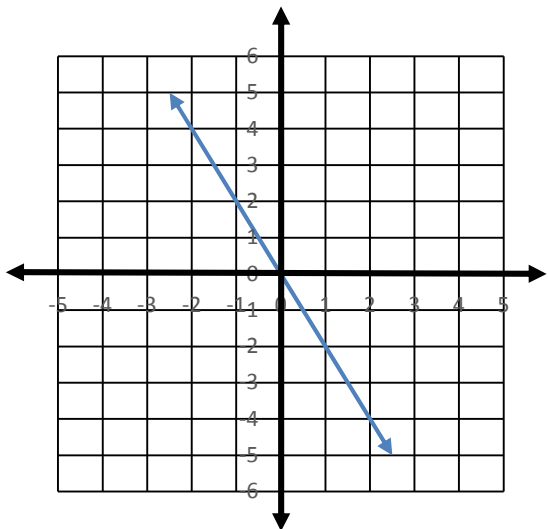| 5. | $(1, 4), (3, 16)$ | | 6. | $(-6, 5), (4, 13)$ | |
|---|---|---|---|---|---|

Erica walks to her friend Philip's house.  The graph shows Erica's distance from home over time.

1. Find the rate of change from 1 minute to 2 minutes.

2. Find the rate of change from 1 minute to 4 minutes.

3. If you know two points on a line, how can you find the rate of change of the variables being graphed?
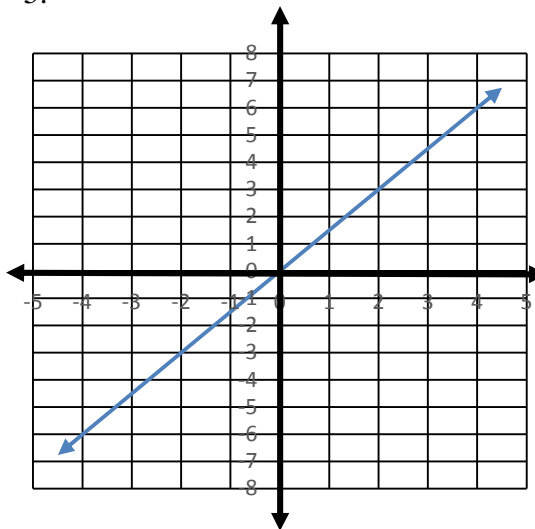


Find the slope of each line.

4.



5.



6. Rectangle EFGH is graphed on a coordinate plane with vertices at
$$E(-3,5), F(6,2), G(4,-4), H(-5,-1)$$

    A. Graph the rectangle above on graph paper.
    B. Find the slopes of each side.
    C. What did you notice about the slopes of opposite sides?
    D. What do you notice about the slopes of adjacent sides?

Find the slope (or rate of change) between each pair of points.

| 7. | $(-3,-4),(-9,-22)$ | | 8. | $(0,3),(8,3)$ | |
|---|---|---|---|---|---|
| 9. | $(2,8),(-6,-8)$ | | 10. | $(0,0),(-3,-4)$ | |
| 11. | $(-1,1),(1,-1)$ | | 12. | $(-10,14),(-7,-1)$ | |
| 13. | $(-6,12),(-9,-3)$ | | 14. | $(1,7),(1,10)$ | |

1. A bicyclist started riding at 8:00AM. The diagram below shows the distance the bicyclist had traveled at different times. What was the bicyclist's average rate of speed in miles per hour?

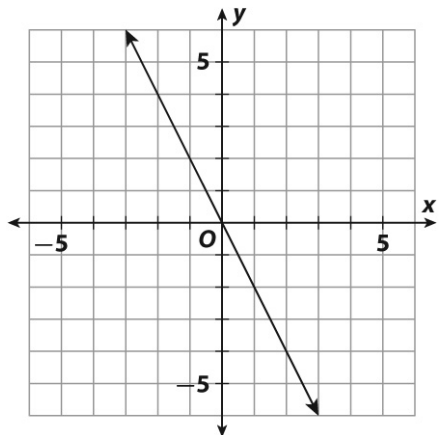8:00AM          4.5 miles          8:18AM          7.5 miles          8:48AM
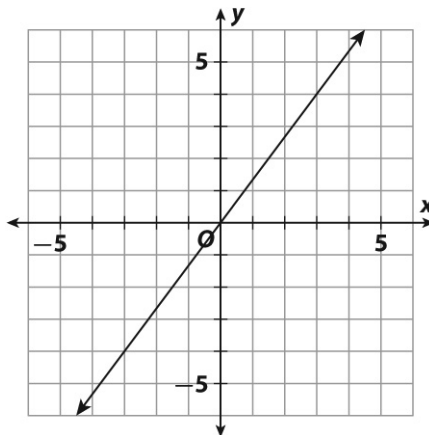
2. A line passes through $(6,3),(8,4),$ and $(n,-2).$ Find the value of $n$.

3. A large container holds 5 gallons of water. It begins leaking at a constant rate. After 10 minutes, the container has 3 gallons of water left.

    A . At what rate is the water leaking?

    B. After how many minutes will the container be empty?

4. Billy found the slope of the line through the points $(2,5)$ and $(-2,-5)$ using the equation $\dfrac{2-(-2)}{5-(-5)}=\dfrac{4}{10}=\dfrac{2}{5}.$ What mistake did he make?

5. Graph parallelogram ABCD on a sheet of graph paper with vertices $A(3,4),B(6,1),C(0,-2),D(-3,1)$.

    A. Find the slope of each side.

    B. What do you notice about the slopes?

    C. Draw another parallelogram on the coordinate plane. Do the slopes have the same characteristics?

6. Ben and Phoebe are finding the slope of a line. Ben chooses two points on the line and used them to find the slope. Phoebe used two different points to find the slope. Did they get the same answer?

7. Two lines pass through the origin. The lines have slopes that are opposites. Compare and contrast the lines.

8. What is the slope of the $x$-axis? Explain.

9. What is the slope of the $y$-axis? Explain.

Find the slope of each line.
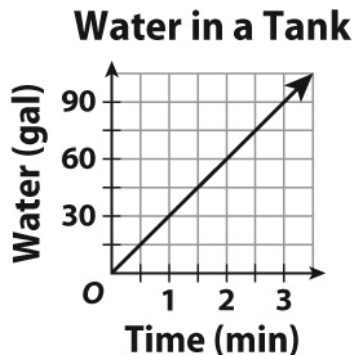
1. slope _____          2. slope _____



Solve.

3. Jasmine bought 7 yards of fabric. The total cost was $45.43. What was the average cost per yard of the fabric she bought?

_____

4. A train traveled 325 miles in 5 hours. What was the train's average rate of speed in miles per hour?

_____

5. The graph at the right shows the amount of water in a tank that is being filled. What is the average rate of change in gallons per minute?

_____

6. Suppose the size of the tank in question 5 is doubled. Will the average rate of change in gallons per minute change? Explain your answer.
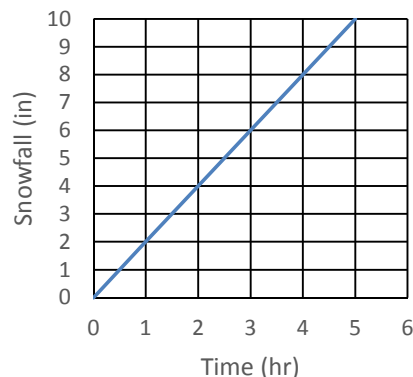


_____

_____

7. A line passes through $(1, 1)$, $(-2, 4)$, and $(6, n)$. Find the value of $n$ algebraically.

_____

A rate is a comparison of two quantities that have different units, such as miles and hours.  A unit rate is a rate in which the second quantity in the comparison is one unit.

A storm is raging on Misty Mountain.  The graph shows the constant rate of change of the snow level on the mountain.

### Misty Mountain Storm

1.  Find the slope of the graph using the points (1,2) and (5,10). Remember that the slope is the constant rate of change.

2.  Find the unit rate of snowfall in inches per hour.

3.  Compare the slope of the graph and the unit rate of change in the snow level.  What do you notice?

4.  Which point on the graph tells you the slope of the graph and the unit rate of change in the snow level?  Explain how you found the point.
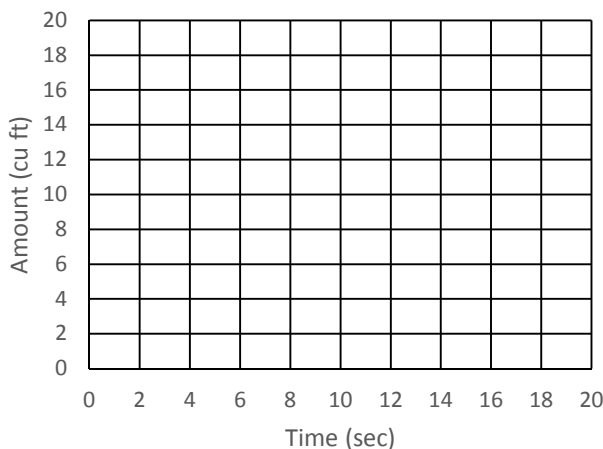
You can use a table and a graph to find the unit rate and slope that describe a real-world proportional relationship.  The constant of proportionality for a proportional relationship is the same as the slope.

5.  Every 3 seconds, 4 cubic feet of water pass over a dam.  Draw a graph of the situation. Find the unit rate of this proportional relationship.
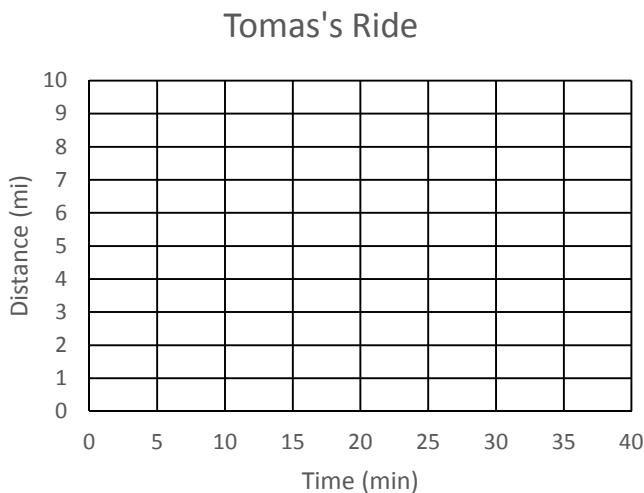
A.  Make a table.

| Time (s) | 3 | | | |
|---|---|---|---|---|
| Volume (ft$^3$) | 4 | | | |

B.  Draw a graph.

C.  Without referring to the graph, how do you know that the point $\left(1, \dfrac{4}{3}\right)$ is on the graph?

1.  Tomas rides his bike at a steady rate of 2 miles every 10 minutes.  Graph the situation.  Find the unit rate of this proportional relationship.
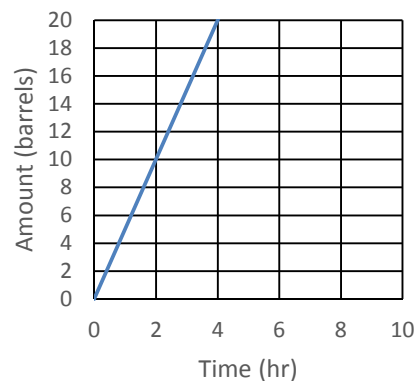
**Tomas's Ride**



You can compare proportional relationships presented in different ways.

2.  The equation $y = 2.75x$ represents the rate, in barrels per hour, that oil is pumped from Well A.  The graph represents the rate that oil is pumped from Well B.  Which well pumped oil at a faster rate?

A.  Create a table for Well A

| Time (hr) | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Quantity (barrels) | | | | |

**Well B Pumping Rate**



B.  Graph Well A on the graph at the right.

C.  What is the unit rate (slope) of Well A?

D.  What is the unit rate (slope) of Well B?

E. Which slope is higher?

F.  Describe the relationships among the slope of the graph of Well A's rate, the equation representing Well A's rate, and the constant of proportionality.
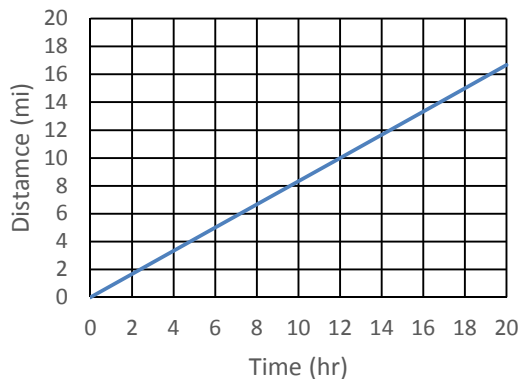
3.  The equation $y = 375x$ represents the relationship between $x$, the time that a plane flies in hours, and $y$, the distance the plane flies in miles for Plane A.  The table represents the relationship for Plane B. Find the slope of the graph for each plane's rate of speed.  Determine which plane is flying at a faster rate of speed.

| Time (hr) | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Distance (mi) | 425 | 850 | 1275 | 1700 |

Give the slope of the graph and the unit rate.

1. Jorge: 5 miles every 6 hours

2. Akiko

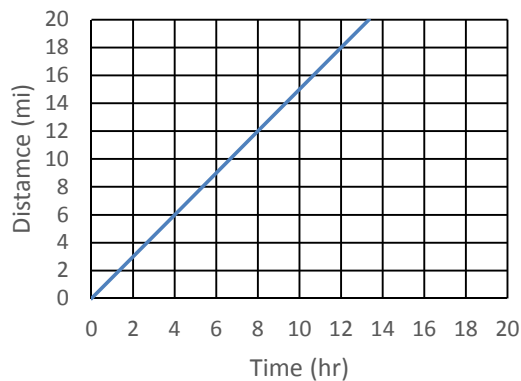| Time (hr)     | 4 | 8  | 12 | 16 |
|---------------|---|----|----|----|
| Distance (mi) | 5 | 10 | 15 | 20 |

Jorge

Akiko

3. The equation $y = 0.5x$ represents the distance Henry hikes in miles over time in hours. The graph represents the rate that Clark hikes. Determine each hiker's slope/unit rate/speed and which hiker is faster.

Clark

Write an equation relating the variables in each table.

4.

| Time (x)     | 1  | 2  | 4  | 6  |
|--------------|----|----|----|----|
| Distance (y) | 15 | 30 | 60 | 90 |

5.

| Time (x)     | 16 | 32 | 48 | 64 |
|--------------|----|----|----|----|
| Distance (y) | 6  | 12 | 18 | 24 |

6. Describe methods you use to show a proportional relationship between two variables, $x$ and $y$. For each method, explain how you can find the unit rate and the slope.
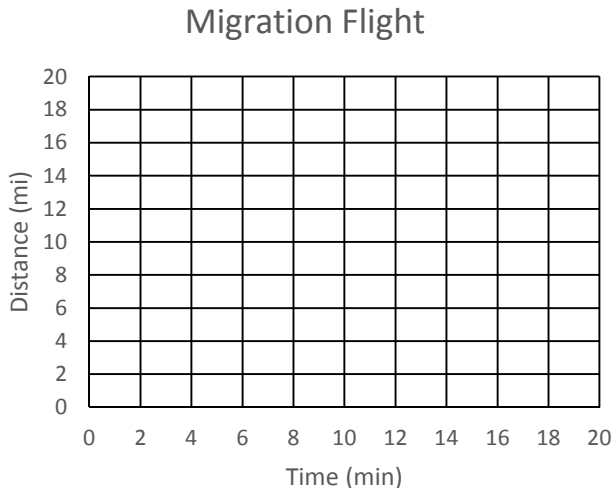
1.  A Canadian goose migrated at a steady rate of 3 miles every 4 minutes.

    A.  Fill in the table to describe the relationship.

| Time (min) | 4 | 8 | | | 20 |
|---|---|---|---|---|---|
| Distance (mi) | | | | | |

    B.  Graph the relationship.

    C.  Find the slope of the graph and describe what it means in the context of this problem.
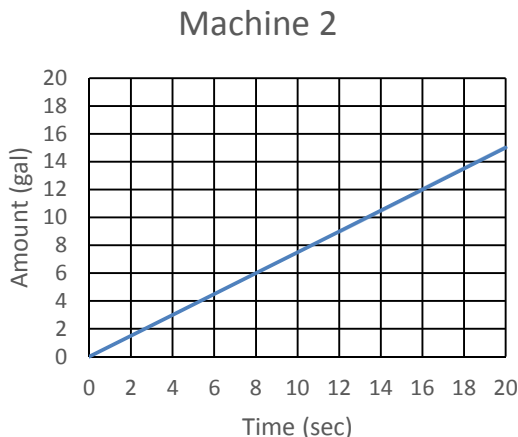
**Migration Flight**



2.  The table and the graph represent the rate at which two machines are bottling milk in gallons per second.

Machine 1

| Time (sec) | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Amount (gal) | 0.6 | 1.2 | 1.8 | 2.4 |

Machine 2

**Machine 2**



    A.  Determine the slope and the unit rate of each machine.

    B.  Determine which machine is working at a faster rate.

1. The equation $y = \dfrac{1}{9}x$ represents the distance $y$, in kilometers, that Patrick traveled in $x$ minutes while training for the cycling portion of a triathlon. The table shows the distance $y$ Jennifer traveled in $x$ minutes in her training. What is each person's training rate and who is faster?
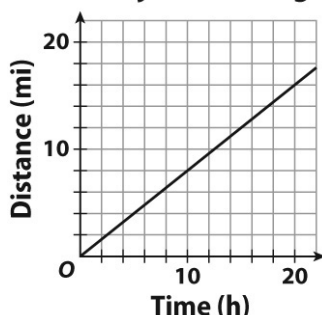
| Time (min) | 40 | 64 | 80 | 96 |
|---|---|---|---|---|
| Distance (km) | 5 | 8 | 10 | 12 |

2. There is a proportional relationship between minutes and cost in dollars. The graph passes through the point (1, 4.75). What is the slope of the graph? What is the unit rate? Explain.

3. Two cars start at the same time and travel at different constant rates. The graph of the distance in miles given the time in hours for Car A passes through the point (0.5, 27.5) and the graph for Car B passes through the point (4, 240). Which car is traveling faster? Explain.

4. The table shows the rate at which water is being pumped into a swimming pool. Use the unit rate and the amount of water pumped after 12 minutes to find how much water will have been pumped into the pool after $13\dfrac{1}{2}$ minutes. Explain your reasoning.

| Time (min) | 2 | 5 | 7 | 12 | 13.5 |
|---|---|---|---|---|---|
| Amount (gal) | 36 | 90 | 126 | 216 | |

For the two questions below find the slope and unit rate.

5.          **Benjamin Hiking**



6.          **Marcy Hiking**

| Time (h) | 5 | 10 | 15 | 20 |
|---|---|---|---|---|
| Distance (mi) | 6 | 12 | 18 | 24 |

7. The equation $y = 3.5x$ represents the rate, in miles per hour, at which Laura walks.

The graph at right represents the rate at which Piyush walks. Determine who walks faster. Explain.
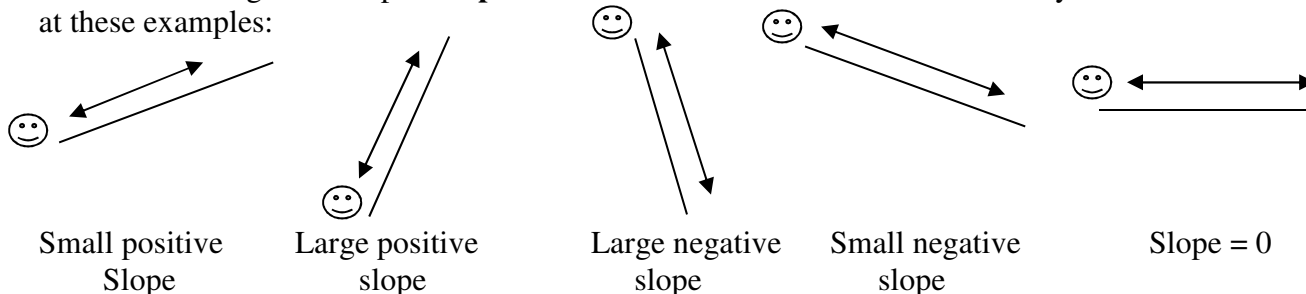
_____

_____

**Piyush Walking**

Which would be a harder ski slope to ski down?

If you said the first one, you have not been skiing before! The second one is much more difficult because it has a greater slope. **Slope** is a mathematical term that we can actually measure. Take a look at these examples:

Small positive        Large positive        Large negative        Small negative        Slope = 0
    Slope                 slope                 slope                 slope

When looking at an equation you can determine whether the slope will be positive or negative using two methods:
1. Graph the equation and look at the direction of your line.
2. Look at the coefficient in front of the *x*, when the equation is written in the form of $y = mx + b$ . If it is a positive number, the slope will be positive. If it is a negative number, the slope will be negative.

Ex.        $y = 2x - 1$        Two is the coefficient, so the slope will be positive.
Ex 2.        $y = -2x - 1$        Negative two is the coefficient, so the slope will be negative.

Determine whether or not the slope of the following lines is **positive** or **negative**.

| 1. | $y = 5x$ | | 2. | $y = 12x - 5$ | |
|---|---|---|---|---|---|
| 3. | $y = -9x + 2$ | | 4. | $y = -x - 17$ | |
| 5. | $y = 5 - 3x$ | | 6. | $y = 4 + 6x$ | |
| 7. | $y = 10 - x$ | | 8. | $y = -54x$ | |
| 9. | $y = \dfrac{1}{5}x - 1$ | | 10. | $y = -0.3x - 3$ | |
| 11. | $y = -5(-2x + 4)$ | | 12. | $y = 3(-2x + 8)$ | |
| 13. | $y = 2(4 + 5 - 2x)$ | | 14. | $y = x$ | |
| 15. | $y = 6$ | | 16. | $x + y = 0$ | |

Deep Flight I is a submarine that can dive to a depth of 3300 feet below sea level at a rate of 480 feet per minute.  Supposed Deep Flight I is going to do a dive starting at sea level.
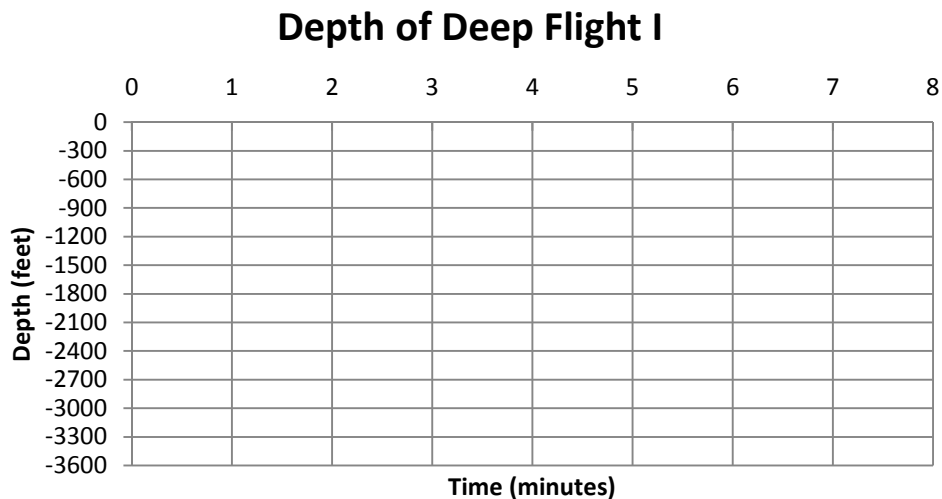
| | |
|---|---|
| 1. | Write an equation to represent the depth of Deep Flight I. |

Use your equation to complete the table below.

| Independent Quantity = Minutes | Dependent Quantity = Depth | Independent Quantity = Minutes | Dependent Quantity = Depth |
|---|---|---|---|
| $x$ | $y$ | $x$ | $y$ |
| 0 | | 4 | |
| 1 | | 5 | |
| 2 | | 6 | |
| 3 | | | |

| | | | | |
|---|---|---|---|---|
| 2. | Why does the table end at 6 minutes? | | | |
| 3. | What do you notice about each depth value in relation to the one before and one after? | | | |
| 4. | In this problem what is the unit rate of change? | | | |
| 5. | How deep would the submarine be after 2.5 minutes?  90 seconds?  45 seconds? | | | |
| 6. | How many minutes would it take Deep Flight I to be 1400 feet below sea level?  2100 feet below sea level?  At its maximum depth? | | | |

Construct a graph of this problem situation.

**Depth of Deep Flight I**

Depth (feet) vs Time (minutes)

x-axis: 0  1  2  3  4  5  6  7  8

y-axis: 0, -300, -600, -900, -1200, -1500, -1800, -2100, -2400, -2700, -3000, -3300, -3600

# PROPORTIONAL RELATIONSHIPS:  CHAPTER 5

**Module 3.1**

When unit rates are equal, that means a constant rate of change.  This is a **proportional relationship.**

**Unit Rate (Ratio)**

The variable $k$ is called the **constant of proportionality**.

A proportional relationship can be written by the equation $y = kx$.

When graphed a proportional relationship is a **straight line** through the **origin**.

In proportional relationships
Unit rate =
Constant rate of change =
Constant of proportionality =
$k$

**Module 3.2**

**Rate of Change =** Amount of change in the output to the amount of change in the input.

In proportional relationships the rate of change is constant.

When graphed the constant rate of change is called **slope**.

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{\text{Change in } y}{\text{Change in } x}$$

**Module 3.3**

Unit Rate

Unit Rate =
Slope =
Constant of proportionality

**Module 3.4**

Constant of proportionality
$k$

A **direct variation** is a relationship that can be written as $y = kx$.

**Constant of variation =**
$k =$
Constant of proportionality

Graphs of direct variation pass through the origin.